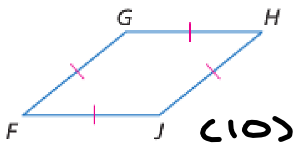
1 Properties of Rhombi and Squares

A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram and the two additional characteristics described in the theorems below.



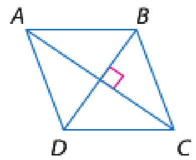
(3) (10)

7 parallelogram + 2 rhombus = 9 total.

Theorems Diagonals of a Rhombus

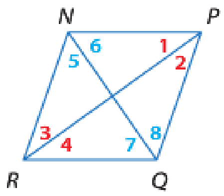
6.15 If a parallelogram is a rhombus, then its diagonals are perpendicular.

Example If ▱*ABCD* is a rhombus, then $\overline{AC} \perp \overline{BD}$.

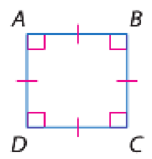


6.16 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

Example If ▱*NPQR* is a rhombus, then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.



A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.



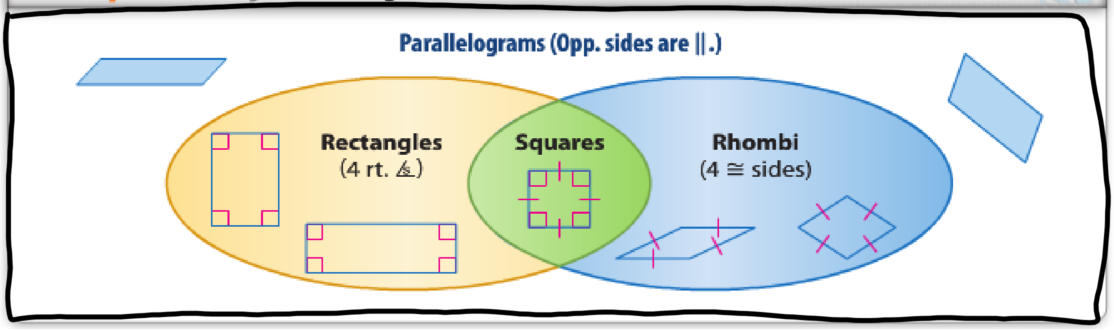
Square *ABCD*

SQUARE = PARALLELOGRAM + RECTANGLE + RHOMBUS.

The Venn diagram summarizes the relationships among parallelograms, rhombi, rectangles, and squares.

SQUARE = 12 total Properties.

Concept Summary Parallelograms

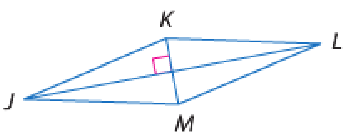


This is assuming you start with a Parallelogram.

Theorems Conditions for Rhombi and Squares

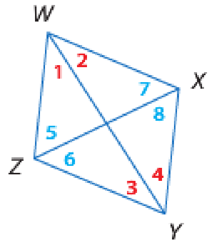
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem. 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then ▱$JKLM$ is a rhombus.



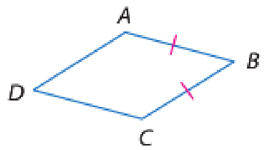
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem. 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then ▱$WXYZ$ is a rhombus.



6.19 If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

Example If $\overline{AB} \cong \overline{BC}$, then ▱$ABCD$ is a rhombus.



6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

****NOTE****

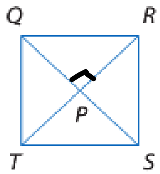
The above conditions ONLY apply if you already know that a quadrilateral is a parallelogram.

EXAMPLE 1: Write a two-column proof.

Given: $QRST$ is a parallelogram.
$\overline{TR} \cong \overline{QS}$, $m\angle QPR = 90$

Prove: $QRST$ is a square.

Rhombus
$\overline{TR} \perp \overline{QS} \rightarrow$ diagonals $\perp$
Rectangle

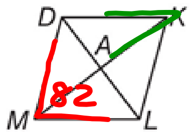


Rectangle + Rhombus = SQUARE

STATEMENTS	REASONS
1. ▱ $QRST$, $\overline{TR} \cong \overline{QS}$ and $m\angle QPR = 90$.	1. Given
2. $QRST$ is a Rectangle	2. $\cong$ diagonals
3. $\overline{TR} \perp \overline{QS}$	3. Def. of $\perp$ lines
4. $QRST$ is a Rhombus	4. $\perp$ diagonals
5. $QRST$ is a square	5. Def. of square

EXAMPLES: 10 properties

ALGEBRA Quadrilateral *DKLM* is a rhombus.



$m\angle DKL = 82$

opp. $\angle$s $\cong$

$m\angle DKM = 41$

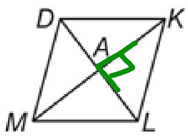
$\frac{82}{2} = 41$

diagonals of rhombus bisect opp. $\angle$s.

2. If $m\angle DML = 82$ find $m\angle DKM$.

EXAMPLES:

ALGEBRA Quadrilateral *DKLM* is a rhombus.



3. If $m\angle KAL = 2x - 8$, find x.

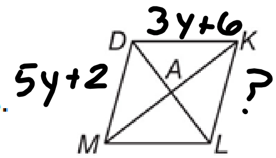
diagonals $\perp$
make right angles

$90 = 2x - 8$
$+8 \quad +8$
$98 = 2x$
$\frac{98}{2} = \frac{2x}{2}$

$x = 49$

EXAMPLES:

ALGEBRA Quadrilateral *DKLM* is a rhombus.



4. If $DM = 5y + 2$ and $DK = 3y + 6$, find KL.

$DM = DK$

$5y+2 = 3y+6$

$-3y-2 \mid -3y-2$

$2y = 4$

$\frac{y}{2}\ \ \frac{}{2}$

$y = 2$

all 4 sides $\cong$

$DM = DK = KL$

$DM = 5(2)+2 = 12$

$10+2$

$KL = 12$